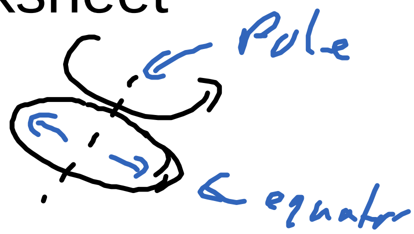
Tuesday, January 12, 2016 8:42 AM

Friday, December 04, 2015
8:45 AM

Gravitational Potential Energy Worksheet solutions

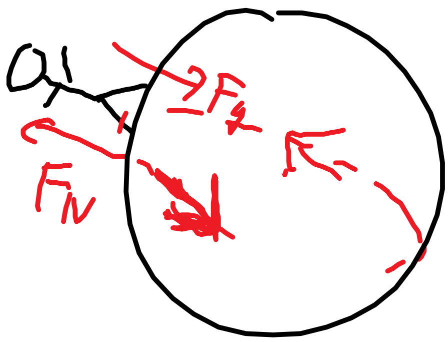


Question one

Because of Earth's rotation

a) the earth bulges in the centre like a spinning water balloon, since F_g is proportional to $1/r^2$, if r increased F_g decreases

b) the spin of the Earth makes you feel slightly less apparent weight because the centripetal acceleration is towards the centre of the Earth, therefore F_N (apparent weight) is less than F_g. (Like being on top of a roller coaster hill, for example. The inertia keeps you moving tangentially to the circular path)
spin at equator causes 0.03m/s^2 variation.
North pole 9.83m/s^2 equator 9.78m/s^2
0.03 by spin, 0.02 by r



$$F_c = F_{net} = F_g - F_N$$

$$\boxed{F_N = F_g - F_c}$$

Question Two

$T=102$ minutes = $102\times60=6{,}120$ s
$g=a_c=4\pi^2r/T^2 = 4\pi^2(3.43\times10^6)/(6120s)^2$
$g=3.62$ N/kg = F_g/m

Question Three
$T=27.3$days = $27.3\times24\times3600=2358720$ s
$g=4\pi^2r/T^2 = 4\pi^2(3.8\times10^8m)/(2358720\ s)^2$
$a=2.7\times10^{-3}\ m/s^2$

Question Four
$a_c=v^2/r=9.80m/s^2$

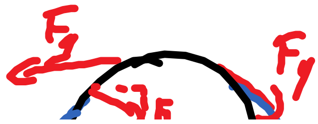
$$v=\frac{\sqrt{(9.80\frac{m}{s^2}\times6.38\times10^6m)}}{\square}$$

$v= 7.9\times10^3m/s$

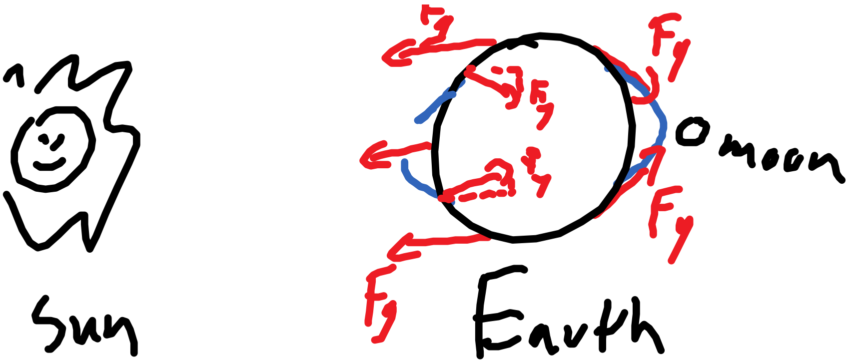
$a = v^2/r = g$

Question Five
$T=365.25$ days = $365.25\times24\times3600=31557600$ s in a year
$g=4\pi^2r/T^2 = 4\pi^2(1.5\times10^{11}m)/(31557600\ s)^2$
$= 5.9\times10^{-3}m/s^2$ you don't feel this as you are in freefall to the Sun.
Note the Earth's acceleration towards the moon is way smaller than towards the Sun.
Why are tides affected by the Moon more than the Sun? https://www.youtube.com/watch?v=pwChk4S99i4

Not to scale



greater



greater tangental component pushing the water in

Six
much smaller $2.7x10^{-3}/5.9x10^{-3}$

Question Seven
a,b,c same
d double

Question Eight
Eg = - GMm/r =
$-6.67x10^{-11}Nm^2/kg^2$ $(5.98x10^{24}kg)$
$(1kg)/6.38x10^6m$
= -6.25×10^7J relative to zero at infinity

Question Nine
solution using Earth's field only
Egf = - GMm/r =
$-6.67x10^{-11}Nm^2/kg^2$ $(5.98x10^{24}kg)$
$(1kg)/3.8 \times 10^8m$
=-1.05×10^6J (so Earth's field = $6.1x10^7J$)
Using the moon's field, set Egi=0
Egf=
$-6.67x10^{-11}Nm^2/kg^2$ $(7.35x10^{22}$ kg)

(1kg)/1.73 x 10^6 m = -2.83x10^6J

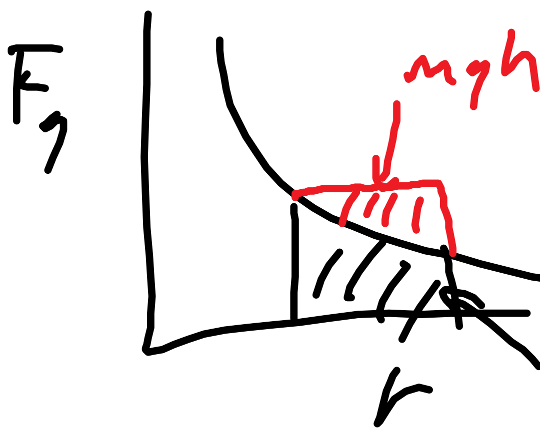
radius of moon

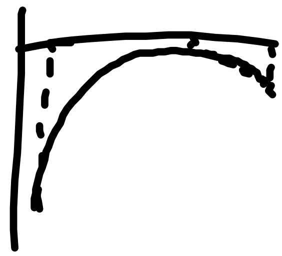
ignore Sun's field

ΔEgt = (Egf+Egf) - Egi

(-2.83x10^6J -1.05 x 10^6J) - (-6.25 x 10^7J)

5.86x10^7J





$= GMm\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$

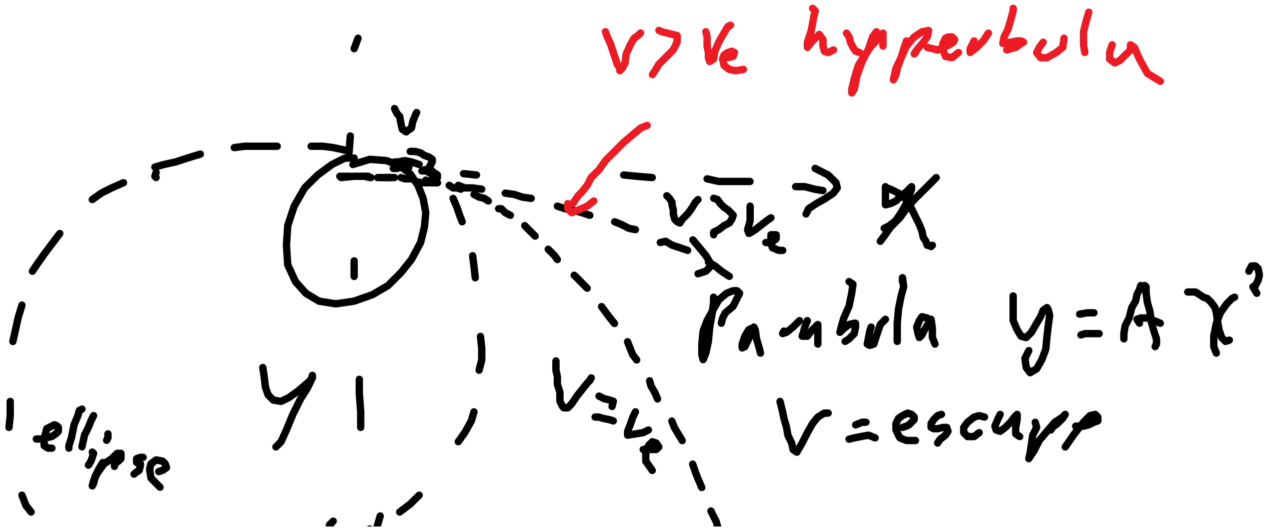
Question ten

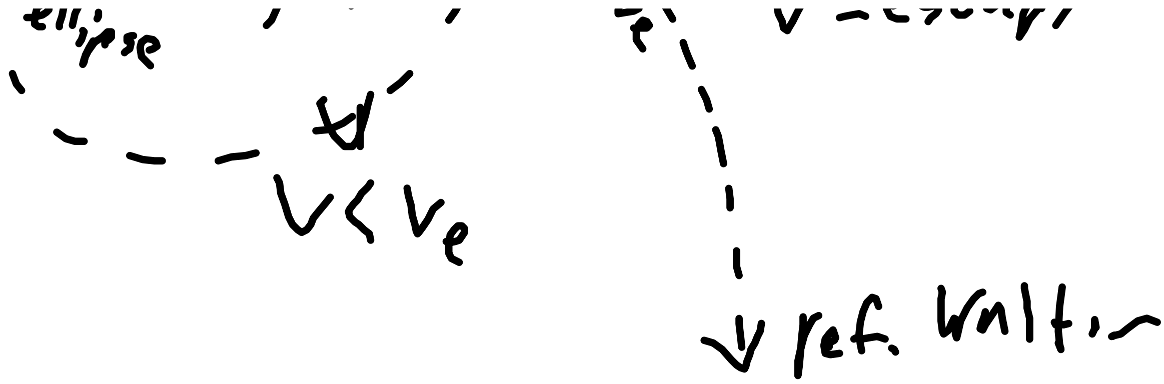
|Eg|=Ek

v= root 2GM/r

root(2 x -6.67x10^{-11}Nm^2/kg^2 (7.35x10^{22} kg)/1.73 x 10^6 m)

= $\underline{2.38 \text{ x} 10^3}$ m/s





Question eleven
Eg Earth

Eg = - GMm/r =
$-6.67x10^{-11}Nm^2/kg^2$ $(5.98x10^{24}kg)$
$(1000kg)/6.38x10^6 = -6.25 \times 10^{10}J$ relative to zero at infinity

Moon
$-6.67x10^{-11}Nm^2/kg^2$ $(7.35x10^{22}$ kg)(1000kg)/1.73 x 10^6 m = $-2.83x10^9J$

ratio = 62.5/2.83=22.0848 times more energy to escape Earth than moon ignoring air resistance

Question Twelve
escape speed = c
$R = 2GM/c^2 = 2(6.67x10^{-11}Nm^2/kg^2)$
$(1.99x10^{30}kg)/(3.00x10^8m/s)^2$
= 2.95 km

Question Thirteen
$E_{binding}$ = GMm/2R

(6.67x10^{-11}Nm^2/kg^2)(1.99x10^{30}kg)(5.98x10^{24}kg)/
2(1.50 x 10^{11}m)
2.65 x 10^{33}J

$$\frac{v^2}{\cancel{r}} = \frac{GM}{r^{\cancel{2}}}$$ orbit speed

$$\frac{1}{2}\cancel{m}v^2 = \frac{GM\cancel{m}}{r}$$ escape speed

$$v^2 = 2\frac{GM}{r}$$